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Standard Cosmology and the BATSE Number vs Peak Flux Distribution

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ABSTRACT

Adopting a simple cosmological model for Gamma Ray Bursts (GRBs), following Mao & Paczyński (1992), we generate number vs. peak flux distributions for a range of values of Ω_0 (ratio of the density of the universe to the critical density) and z_{\max} (the redshift at which the faintest GRB in the present sample is located), and compare these distributions to one from BATSE GRBs in the 2B catalog. The observed BATSE distribution is consistent with the faintest GRBs in our sample originating from a redshift of $z_{\max} \sim 0.8-3.0$ (90%), with the most likely values in the range of 1.0-2.2, and is largely insensitive to Ω_0 for models with no evolution.

To constrain the model parameter Ω_0 to the range 0.1-1.0 using only $\log N - \log F_{\text{peak}}$ distributions, more than 4000 GRBs, with a most likely value of $\sim 9,000$ GRBs, above the 1024 msec averaged peak flux of $0.3 \text{ phot cm}^{-2}\text{sec}^{-1}$ would be needed. This requires a live integration time of > 6 years with BATSE. Detectors sensitive to much lower limits (*i.e.* standard candle bursts out to $z_{\max}=10$, $\sim \times 70-400$ in sensitivity) require ~ 200 GRBs, with < 0.03 year 4π ster coverage.

We place limits on the amount of frequency density and, separately, peak luminosity evolution in the sample of GRBs. We find that frequency density evolution models can place the faintest GRBs at $z_{\max} \sim 10-200$, without conflicting with the observations of relative time dilation of ~ 2 reported by Norris et al. (1994a) and Wijers & Paczyński (1994) (however, see Band 1994), although this would require vastly different comoving burst rates in GRBs of different spectra.

1. Introduction

More than 20 years after their discovery (Klebesadel, Strong, & Olson 1973), Gamma Ray Bursts (GRBs) continue to be elusive about their origin or, possibly, origins.

A number of workers report results which are consistent with the cosmological hypothesis, using data obtained by the Burst and Transient Source Experiment (BATSE; Fishman et al. 1989) on the *Compton* Gamma Ray Observatory (CGRO).

A small subset of 'simple' single-peaked GRBs observed by BATSE showed that bursts with the lowest peak fluxes had longer rise times than GRBs with the highest peak fluxes (Kouveliotou et al. 1991).

Dermer (1992) found that a cosmological model with no source evolution and a $q_0=1/2$ Friedmann cosmology could reproduce the differential $\langle V/V_{\max} \rangle$ distribution from 126 BATSE GRBs. It was also found that the observed $\langle V/V_{\max} \rangle$ could accommodate moderate frequency density evolution, with $(1+z)^p$, for $p = -2, 0$, and 2 .

Piran (1992) found the BATSE-observed $\langle V/V_{\max} \rangle$ statistic (see, *e.g.*, Schmidt, Higdon, & Hueter 1988) and number distribution $N(V/V_{\max})$ are consistent with two flat-space cosmological models, one with a zero cosmological constant and mass density $\Omega_M=1$, and a second with $\Omega_M=0.1$ and a non-zero cosmological constant (Λ), with the faintest GRBs coming from $z_{\max}\sim 1$. It was also found that the $\langle V/V_{\max} \rangle$ distribution could accommodate some frequency density evolution, with $(1+z)^p$, for $p = -3$ to $3/2$.

Mao & Paczyński (1992; hereafter MP), assuming no evolution in the burst rate (a constant frequency density), and an $\Omega_0=1$, $\Lambda = 0$ universe, found that BATSE GRBs with assumed photon number spectral indices in the range 0.5–1.5 (somewhat flatter than actually observed by BATSE) would match the $\langle V/V_{\max} \rangle$ value from the first ~ 150 GRBs observed by BATSE (Meegan et al. 1992) if the faintest GRBs originated from $z_{\max}=1.1$ –2.6.

Lestrade et al. (1993), following a suggestion by Paczyński (1992), used 20 GRBs observed by the PHEBUS instrument on board GRANAT with durations < 1.5 sec. They showed that stronger bursts (in peak counts above background in a 1/4 sec time bin) had a significantly shorter average duration than weaker bursts. Using a Kolmogorov-Smirnov (KS) test, a 97% probability that the stronger GRBs were drawn from a different duration population than the weaker GRBs in this sample was found.

Wickramasinghe et al. (1993) fit the $\log N - \log F_{\text{peak}}$ curve of 118 BATSE GRBs with a model of GRBs of cosmological origin, assuming a constant number of burst sources per comoving volume – a different assumption than that made by MP – which does not take into account the effect of time-dilation on the burst rate. It was found that GRBs at the completeness flux limit could originate at $z_{\max}=0.4$ –1.7, depending on the assumed source spectrum (photon number spectrum in the range of 1.5–2.5; softer spectrum produced higher z_{\max} values).

Davis et al. (1993), using an analysis of pulse-width distributions from 135 BATSE GRB lightcurves found that dim GRBs have pulse-widths that are of ~ 2 greater than that of bright bursts, consistent with the dimmest GRBs coming from a $z\sim 1$.

Tamblyn & Melia (1993) found that, assuming a “standard” energy-spectral break at 300 keV, the average value of ratio of the flux detection limit to the peak burst flux to the $3/2$ power ($\langle (F_{\min}/F_{\max})^{3/2} \rangle$) as a function of the observed full-sky burst rate for several different GRB instruments (*i.e.* PVO, SMM, KONUS, APEX, SIGNE, and BATSE) is consistent with a non-evolving cosmological population.

Fenimore et al. (1993), using a composite intensity distribution of BATSE and Pioneer Venus Orbiter (PVO) experiments, fit a homogeneous model of standard candle sources, with the faintest BATSE bursts coming from $z \sim 0.80$; using model-derived distances to the brightest sources within the error boxes of 8 bright GRBs, they found that host galaxies must be fainter than an absolute magnitude of -18 .

Lamb, Graziani & Smith (1993) found that bursts which are “smooth” on short (~ 64 ms) timescales have peak counts integrated on a 1024 msec timescale (B) which indicate “smooth” bursts are both faint (*i.e.* low number of counts) and bright (*i.e.* high number of counts), while bursts which are “variable” on short timescales are faint only. The “smooth” and “variable” classification is based on a parameter V , the ratio of

the peak counts in a 64 msec time bin to the peak counts in a 1024 msec time bin ($C_{\text{max}}^{64}/C_{\text{max}}^{1024}$), measured by the second most brightly illuminated Large Area Detector of BATSE. There is a known systematic correlation of V with burst duration (Rutledge & Lewin 1993); the minimum V value is a function of duration t_{dur} , the result of which is that all GRBs of $t_{\text{dur}} \lesssim 400$ msec are included in the “variable” class. These short GRBs make up $\sim 2/3$ of the “variable” class.

It might thus appear that the conclusion of LGS requires that short GRBs be faint, while long GRBs be both bright and faint, in apparent contradiction to the conclusions of Lestrade et al. (1993). However, since the durations of these short, “variable” bursts are less than the sampling period, B represents a measure of counts-fluence, rather than a sampled instantaneous peak flux. Thus, one should not conclude that the results of LGS are in contradiction to those of Lestrade et al. (1993), (specifically, that shorter GRBs have higher peak fluxes than longer GRBs, consistent with the time dilation effect predicted by Paczyński 1992).

Norris et al. (1994a), using three tests to compare the time-scales of 131 bright and dim GRBs observed by BATSE of duration greater than 1.5 sec, finds temporal, intensity, and fluence scaling effects consistent with a cosmological time dilation, with results consistent with the dimmest GRBs coming from a $z \sim 1$.

Wijers & Paczyński (1994) analyzed the time dilations of bright and faint samples of bursts, and found the relative duration distribution widths were the same. They concluded that the effect is more convincingly explained by a cosmological origin than by a distribution intrinsic to the source population.

A cosmological source of GRBs is not necessarily the sole explanation of the results of these analyses. It cannot be ruled out that the results are consistent with the intrinsic characteristics of a population located in an extended galactic halo (or, corona). Hakkila et al. (1994) recently produced a detailed study of the constraints on galactic populations from BATSE observations.

Here, we use the cosmological model for GRBs suggested by MP, except we adjust Ω_0 and z_{max} to find $\log N - \log F_{\text{peak}}$ distributions consistent with BATSE observations, for a range of assumed energy spectra, in order to set limits on the acceptable parameter space for non-evolving cosmological models, and for cosmological models with evolving frequency density and luminosity. We calculate the number of GRBs which would be required to differentiate between values of Ω_0 of 0.1 and 1, and also between values of 0.1 and 0.5, depending on the sensitivity of the detector as indicated by z_{max} . Finally we calculate the amount of integration time required to distinguish between these two values of Ω_0 .

2. Analysis

Throughout this paper, we assume a cosmological constant $\Lambda=0$, and a universe dominated by non-relativistic matter, which sets $\Omega_0 = 2q_0$, and spherical symmetry of radiation (no beaming). The effects of beaming on GRB observations have been considered elsewhere (Mao & Yi 1994).

We also make extensive use of the Kolmogorov-Smirnov test for comparing an observed distribution with a model distribution (*cf.* Press et al. 1988, p. 491). This test results in a probability that, given the model distribution, another observed distribution as disparate or more disparate could be drawn from the source population. Thus, a KS probability of 1% implies that only 1% of the data sets of the same size as the observed data set would be as or more disparate from the model as the observed data set. Subsequently,

when we say “90% confidence limits”, we mean the boundary around those models with KS probabilities >10%.

2.1. Number- Peak Flux distribution

This discussion follows closely the model described by MP; we have assumed their “simplest” model, with no adjustable parameters, except we adjust one of the parameters (Ω_0).

2.1.1. The Assumed Spectrum

We here discuss our choice of the assumed GRB spectrum, as this effects our results quantitatively, although not qualitatively, and warrants clarification.

We first assume that all GRBs have identical peak luminosities (*i.e.* are standard candles), with (first) identical power law photon number spectra:

$$\frac{dI_\nu}{d\nu} = C_z \nu^{-\alpha} \quad (1)$$

$$C_z = C_0(1+z)^l \quad (2)$$

where I_ν is in the units photons sec^{-1} , and C_z is a z-dependent peak luminosity normalization, which accounts for the peak luminosity evolution with redshift. We chose to represent the evolution by power-law for its simplicity, unmotivated by any theoretical expectation that it should actually follow a power-law form.

The exponent α has been found to be largely in the range of 1.5–2.5 (Schaefer et al. 1994) for 30 GRBs selected from among 260 GRBs in the first BATSE catalog (Fishman et al. 1994). We are motivated to use a simple power-law spectrum, as a simple power-law spectrum (of $\alpha=2.0$) was assumed in producing the observed peak photon flux values in the BATSE 1B catalog. Schaefer et al. (1994) found that single power-law spectra adequately describe the observed GRB spectra for the majority of bursts. Using data from Table 5 Schaefer et al. (1994) we produced the number histogram (Fig 1) of the 30 measured α , for fits over all “valid energy ranges”. The majority of these GRBs have α between 1.5 and 2.5. In fits over energy ranges < 500 keV, the resultant values of α tend to be harder. It should be noted that the GRBs in this sample were selected for their high statistics, and it is known that GRBs which have higher peak fluxes are spectrally harder than bursts with lower peak fluxes (Norris et al. 1994a).

The single power-law energy spectrum is an over-simplification of the true GRB energy spectrum. Band et al. (1993) found that GRB energy spectra are well described at low energies by a power-law with an exponential cutoff, and at high energies by a simple (steeper) power-law, of the form:

$$\frac{dI_\nu}{d\nu} = C_z \nu^{-\alpha_b} \exp^{-\nu/\nu_b} \quad \nu < \nu_b(\alpha_b - \beta) \quad (3)$$

$$= C_z \nu_b^{-\alpha_b} \exp^{-1} \nu^{-\beta} \quad \nu > \nu_b(\alpha_b - \beta) \quad (4)$$

where C_z is as in Eq. 2 (here, we use α_b to differentiate between this power law and the single power-law parameter α). In Fig. 2a, we show the fit spectral models for the ~ 55 bursts produced by Band et al. (1993) (their Table 4), normalized at 50.0 keV, over the energy range 50-5000 keV. As Band et al. concluded, there is no “universal” spectrum, and this complicates interpretation of analyses which rely on a single assumed “universal” spectrum, such as has been performed by Tamblyn and Melia (1993). However, a spectrum which largely follows the behavior of the burst spectra, has the values $\alpha_b=1.0$, $\beta = 2.5$, and $h\nu_b = 300$ keV (hereafter, the complex spectrum). Further, the majority of these spectral models are steeper than a single power-law $\alpha = 1.0$, but more shallow than a single power-law $\alpha = 2.5$. We show these three models in Fig. 2b; the complex spectrum lies at all times between the single power-law spectra $\alpha = 1.0$ and 2.5.

Our approach here is to perform the analyses separately for two single-power law spectral slopes ($\alpha=1.0$, and 2.5) and for the above-defined complex spectrum. Thus, the acceptable parameter space of cosmological models will be conservatively defined by the acceptable parameter space spanned by models for each of the assumed energy spectra. Our use of single power-law spectra will be justified by the qualitatively similar behavior between model parameter space using the complex spectra and that using single power-law spectra.

2.1.2. Procedure for Single Power-law Spectra

We assume the burst detectors are sensitive only to photons within a fixed frequency range ($\nu_1 \leq \nu \leq \nu_2$). For a source with a single power-law photon spectrum, located at redshift z , the part of the spectrum which is shifted into the detector bandwidth is:

$$I(z) = \int_{\nu_1(1+z)}^{\nu_2(1+z)} \frac{dI_\nu}{d\nu} d\nu = I_z(1+z)^{-\alpha+1} \quad (5)$$

$$I_z = (1+z)^l \int_{\nu_1}^{\nu_2} C_0 \nu^{-\alpha} d\nu \quad (6)$$

$$I_z = I_0(1+z)^l \quad (7)$$

Thus, $I(z)$ is the peak luminosity (photons sec^{-1}) measured at redshift z which will be in the detector bandwidth at $z=0$. I_z includes the term $(1+z)^l$, which we use to make an accounting of evolution in the peak luminosities of GRBs as a function of redshift, and I_0 is a constant. This form of evolution is here adopted as a convenience, and is not motivated either by observations of evolving populations (*e.g.* radio galaxies, faint blue galaxies) nor by theoretical predictions. This limits the usefulness of the conclusions to describe only those models which can be accommodated by power-law evolution over the range of redshifts the model implies GRBs are observed by BATSE.

To find the measured luminosity, we must correct for time dilation, by dividing $I(z)$ by an additional redshift factor $(1+z)$. The number of photons measured at redshift $z=0$ will be equal to the photon luminosity at $z=0$, multiplied by the ratio (f) of the solid angle subtended by the detector (of area A) to the total solid angle (4π ster) (Weinberg 1972, p. 421):

$$f = \frac{A}{4\pi R_0^2 r_z^2} \quad (8)$$

This measured flux (in $\text{phot cm}^{-2} \text{ sec}^{-1}$) is:

$$F(z) = \frac{I_0 (1+z)^{-\alpha+l}}{4\pi R_0^2 r_z^2} \quad (9)$$

where I_z is in photons sec^{-1} and r_z , the co-ordinate distance, is (from Weinberg 1972, p. 485):

$$r_z = \frac{zq_0 + (q_0 - 1)(\sqrt{2q_0z + 1} - 1)}{H_0 R_0 q_0^2 (1+z)} \quad (10)$$

where q_0 is the deceleration parameter, H_0 is the Hubble parameter at the current epoch, R_0 is the expansion parameter at the current epoch.

We find the total number of GRBs observable out to a redshift z' during an observation period of Δt at the present epoch:

$$N_b(z') = \int_0^{z'} \frac{\Delta t}{1+z} 4\pi n_z r_z^2 \frac{dr_z}{dz} \frac{dz}{\sqrt{1-kr^2}} \quad (11)$$

where $k = -1, 0$, or 1 for an open, critical, and closed universe, and n_z is the number of GRBs per comoving volume per comoving time (frequency density) as a function of redshift z , accounting for evolution, which we take simply to be of a law form:

$$n_z = n_0 (1+z)^p \quad (12)$$

with n_0 a constant. When $p=0$, the model is one of no evolution; when $p>0$, then we have “positive” evolution – an increasing comoving burst rate with increasing redshift – and when $p<0$ we have “negative” evolution – a decreasing comoving burst rate with increasing redshift. Eq. 11 can be rewritten as:

$$N_b(z') = \Delta t \frac{4\pi n_0}{q_0^4 (R_0 H_0)^3} \int_0^{z'} \frac{(1+z)^{p-4} (zq_0 + (q_0 - 1)(\sqrt{1+2q_0z} - 1))^2}{\sqrt{1+2q_0z}} dz \quad (13)$$

Using these assumptions, we perform the following analysis. We define a sample of M GRB peak fluxes, taken from the second BATSE catalog (Fishman et al. 1994), from bursts which have a peak flux measured on the 1024 msec time scale greater than $0.3 \text{ photons cm}^{-2} \text{ sec}^{-1}$ in the 100-300 keV band, the peak flux to which the trigger efficiency is confidently known (C. Meegan, private communication). We find 394 bursts which meet these criteria.

We sort the peak fluxes in increasing order and produce an observational integrated number – peak flux curve. We correct the observational number – peak flux curve for the BATSE trigger efficiency table, given in the second catalog. We use an analytic approximation to the trigger efficiency of the form:

$$\text{efficiency} = A - B \exp \frac{C - F_{\text{peak}}}{D} \quad (14)$$

$$\begin{aligned} A &= 0.9989 \\ B &= 0.003933 \\ C &= 0.6496 \\ D &= 0.0748 \end{aligned}$$

and F_{peak} is the peak flux measured by BATSE. This equation is accurate to better than 1.2%, which is sufficient accuracy for the burst samples sizes considered here.

We assume a value for $\Omega_0 (=2q_0)$, which sets the functional dependence of r_z on z (eqn. 10). We then:

1. assume a value for z_{\max} , the redshift from which the burst with lowest observed peak flux was emitted ($F(z_{\max})$), and numerically integrate eqn. 11 to find $N_b(z_{\max})$;
2. set $N_b(z_{\max})=M$ (here, 394). This sets the value n_0 as a function of assumed z_{\max} and q_0 , with knowledge of H_0 .

Then, for each burst, ordered in increasing peak flux, we do the following:

1. from eqn. 9, we find the ratio of burst peak flux to the minimum peak flux of a detected GRB to be:

$$\frac{F(z)}{F(z_{\max})} = \left(\frac{r_{z_{\max}}}{r_z} \right)^2 \left(\frac{1+z}{1+z_{\max}} \right)^{-\alpha+l} \quad (15)$$

This sets the value z at which the burst flux $F(z)$ was measured. The ratio of r_z with $r_{z_{\max}}$ is independent of n_0 , H_0 and R_0 – thus, dependent only on z , α , and l .

2. Using z in eqn. 11, we find $N_b(z'(F))$, which permits us to produce a theoretical $\log N - \log F_{\text{peak}}$ distribution, for an assumed z_{\max} , α , and Ω_0 , l , and p .

After producing a theoretical $\log N - \log F_{\text{peak}}$ distribution as we describe, we compare this to the observed distribution, using a single distribution KS test, to find the probability that the observed distribution was produced by the model distribution.

This was done for 3 values of α , and for $0.1 \leq z_{\max} \leq 955$ at points separated by $\Delta \log(z_{\max})=0.041$, and for $10^{-2} \leq \Omega_0 \leq 2$, separated by $\Delta \log(\Omega_0)=0.041$.

The calculation was performed for the case of no frequency density or peak luminosity evolution ($p=0, l=0$), and separately for frequency density evolution (integer values of p between -4 and 4, inclusive) and peak luminosity evolution (integer values of l between -4 and 4, inclusive).

2.1.3. Procedure for Complex Spectrum

The procedure to fit the $\log N - \log F_{\text{peak}}$ curve for the assumed complex spectrum is similar to that followed above, differing only in detail. The single power-law spectrum is responsible for the simple analyticity of eqn. 15, the ratio of the peak fluxes of standard candle bursts at different redshifts. The complex spectrum requires that the ratio be found numerically. A further systematic complication is that, as the origin of the spectral “break” is not well understood, exactly at what co-moving energy the spectral break should be placed is not obvious, although the observational limit that the vast majority of the spectral breaks have $E_b < 400$ keV (Band et al. 1993) provides an important guide. We have made the choice to set the co-moving breaking energy for each simulation so that the observed breaking energy is at the top of the passband of the 2B catalog peak fluxes (300 keV) for a burst at the *implied* lowest redshift z_{\min} (that is, for the brightest burst). Thus, for a model with z_{\max} which requires a certain z_{\min} , then the assumed spectrum has $E_b=300(1+z_{\min})$ keV (found iteratively) and therefore for all fainter bursts the E_b (observed at earth) will lie between $300 - 300(1+z_{\min})/(1+z_{\max})$ keV, as breaking energy is held constant. We discuss the implications of this choice further in Sec. 4.

As in Eq. 9, the measured flux for the complex spectrum burst is:

$$F(z) = \frac{\int_{\nu_1(1+z)}^{\nu_2(1+z)} \frac{dI_\nu}{d\nu} d\nu}{4\pi R_0^2 r_z^2} \quad (16)$$

where the differential photon spectrum is given in Eqs. 3-4, and the values $\alpha_b=1.0$ and $\beta=2.5$ are adopted. Thus, the ratio of burst peak flux to the minimum peak flux of a detected GRB is (replacing eq. 15):

$$\frac{F(z)}{F(z_{\max})} = \left(\frac{r_{z_{\max}}}{r_z} \right)^2 \left(\frac{\int_{\nu_1(1+z)}^{\nu_2(1+z)} \frac{dI_\nu}{d\nu} d\nu}{\int_{\nu_1(1+z_{\max})}^{\nu_2(1+z_{\max})} \frac{dI_\nu}{d\nu} d\nu} \right) \quad (17)$$

The value of E_b ($=h\nu_b$; Eq. 3 & 4) is found by iteratively $F(z_{\min})/F(z_{\max})$ (=ratio of the highest to lowest observed GRB peak flux) until $E_b/(1+z_{\min}) = 300$ keV. The integral is evaluated with the limits $h\nu_1 = 50$ keV and $h\nu_2 = 300$ keV (the passband in which the peak flux is measured in the 2B catalog).

Once the value of E_b is set, the analysis proceeds using Eq. 17 to find the value z at which the burst flux $F(z)$ was measured.

2.2. Observational confrontation with cosmological model

An essential parameter to this model is Ω_0 . Using the zero-evolution and “standard candle” luminosity assumptions, we estimate how many GRBs are required to differ between parameter values of Ω_0 with a KS probability of 1%, assuming a detector which is sensitive to GRBs out to a redshift z_{\max} . Our motivation for doing so is that this parameter is the least constrained by the present analysis of those in the considered model. A calculation of how many GRBs are required to constrain the parameter, based on these very (and probably, overly) simple assumptions, would set an upper limit to the number of GRBs required for the intrinsic characteristics of the source population (*e.g.* luminosity function, frequency density evolution, luminosity evolution) to manifest themselves.

We wish to estimate the number of GRBs (N) required to discern between two values of Ω_0 , assuming standard candle GRBs of constant number per comoving time per comoving volume. To do this, we assume a z_{\max} , the redshift at which the standard candle GRBs produce a flux at the completion limit.

We produce $\log N - \log F_{\text{peak}}$ distributions for $\Omega_0=1.0$ and $\Omega_0=0.1$, assuming a constant number of bursts per comoving time per comoving volume, sampled at 200 points separated by a constant amount of comoving time-volume. The normalized distributions are the cumulative probability distributions, and are not a function of the number of detected bursts. We compare the two distributions using a single distribution KS test.

For those cases when the number of required GRBs was less than the sampling of the cumulative probability distribution, the process was repeated, with the cumulative probability distributions sampled, instead of with 200 GRBs, with N GRBs found in the first iteration.

We perform the same analysis to estimate the number of GRBs required to discern between values of $\Omega_0=0.5$ and $\Omega_0=0.1$.

3. Results

3.1. Number – Peak flux distribution: no evolution

The results of fitting the peak photon count rate number distributions to model populations with no evolution ($p=0$, $l=0$) are shown in Fig. 3. Each panel indicates the spectrum assumed (“complex” or single power-law $\alpha=1.0$, 2.5). The lines are constant KS probability (1%, 10% and 33%) that the modeled number-peak flux distribution produces a distribution as or more disparate than the observed distribution.

The most probable values of z_{\max} , evaluated at ($\Omega_0=1$) are ~ 1 – 2.2 (depending on the assumed spectrum). For all spectra considered, the 90% confidence limits on z_{\max} are 0.8–3.0 (at $\Omega_0=1$) and are largely insensitive to the assumed value of Ω_0 . As these confidence ranges assume the source population to be entirely of a single spectral type, the confidence range is systematically overestimated.

As expected, the behavior of the complex spectral model as a function of (Ω_0 , z_{\max}) is qualitatively similar to that of the single power-law spectra models, and the resulting acceptable parameter space is between that of the $\alpha = 1.0$ and 2.5 models. We therefore proceed with confidence that the behavior of the two single power-law models will place conservative limits on the behavior of the complex spectrum model. It is desirable to do so only because of the computational demands of the complex spectral model.

Each of the comparisons of theoretical curves with the observed curves results in a value of the constant n_0 . This value is the necessary normalization to obtain the number of observed GRBs M during the observation period out to z_{\max} , from Eq 13.

Following the procedure used by MP (Eq 13-15), we renormalize n_0 to n_* , the number of GRBs per L^* galaxy per 10^6 years at the current epoch. In Fig. 3, the lines marked “10”, “1.0”, and “0.1” indicate n_* in units of GRB per L^* galaxy per 10^6 yr. For all spectra considered, the value of n_* is conservatively in the range 0.3–5.0 per L^* galaxy per 10^6 yr at $\Omega_0=1$ (99% confidence), consistent with values found by other investigators (Dermer 1992; MP; Wickramasinghe et al. 1993).

For high z_{\max} , one might reasonably expect that there is evolution in the luminosity density of galaxies. The re-normalization (to n_*) we adopted here is thus pertinent only for redshifts where such evolution can be neglected. To change the values back to burst rate per comoving volume at the present epoch, one multiplies by the galaxy luminosity density we used (from Efstathiou, Ellis & Peterson 1988):

$$n_0 = n_* \times \phi^* \Gamma(2 + \alpha_g) \quad (18)$$

$$\phi^* \Gamma(2 + \alpha_g) \sim 1.67 \times 10^{-2} L^* h_{100}^3 \text{Mpc}^{-3} \quad (19)$$

As this re-normalization is multiplication by a constant, the *relative* burst frequency densities do not change; thus n_* may be used to compare frequency densities beyond redshifts where evolution becomes important, with the assumption that at such redshifts the burst frequency densities are unrelated to galaxy evolution.

3.2. Observational confrontation with cosmological model

As is evident in Fig. 3, the Number distributions are largely insensitive to the value of Ω_0 , for the 394 GRBs in this sample.

In Fig. 4a & b, we show the number of GRBs which would be required to discern between $\Omega_0=1$ and 0.1, & $\Omega_0=0.5$ and 0.1, from the number-peak flux distribution, using data which is complete for standard candle GRBs out to a redshift of z_{\max} .

The points are the results of the calculation at a few values of z_{\max} , assuming different spectra ($\alpha=1.0$ or 2.5); the lines between the points are there to “guide the eye”, and also identify the spectral slope assumed for the calculation. The solid line is $\alpha=1.0$ and the dashed line is $\alpha=2.5$.

The most likely z_{\max} for BATSE GRBs calculated in the previous section was (2.2, 1.0) for $\alpha = (1.0, 2.5)$. For both spectra, this corresponds to $\sim 9,000$ required for BATSE to discern between $\Omega_0=1$ and 0.1. Similarly, approximately 20,000 GRBs are required to discern between $\Omega_0=0.5$ and 0.1.

The uncertainties on these values are large. Permissively, if the actual z_{\max} value is 3.0 (90% upper limit for $\alpha=1.0$), then only $\sim 4,000$ (or 10,000) GRBs are required to discern between $\Omega_0=1$ and 0.1 (or 0.5 and 0.1).

The number of GRBs required diminishes by 4 orders of magnitude when z_{\max} changes from 0.5 to 10. For $\Omega_0=1.0$, a change in depth of $z_{\max}=1.0$ to 10.0, requires an increase in detector sensitivity of a factor of (30, 400) for $\alpha=(1.0, 2.5)$.

In Fig. 4c & d, we show the coverage (in years $\times 4\pi$ ster) required for a detector complete to burst peak fluxes out to z_{\max} to obtain the number of GRBs required to discern between different values of Ω_0 in the $\log N - \log F_{\text{peak}}$ distribution, assuming a constant number of GRBs per comoving time, per comoving volume. We normalize to 394 GRBs detected with peak flux > 0.3 phot $\text{cm}^{-2} \text{sec}^{-1}$ on the 1024 msec integration timescale in the 2.7×10^8 sec ster coverage of the second BATSE catalog, which introduces $\sim 5\%$ uncertainty in the coverage.

At the minimum, BATSE (assuming 2.6π ster coverage) would require $\gtrsim 6$ years integration time before the parameter Ω_0 becomes reasonably constrained by the Number – peak flux distribution to permit confrontation the cosmological models using only the distribution. The most likely required integration time is ~ 30 years. Detectors sensitive out to $z_{\max}=10$ would give statistically different Number-peak flux distributions after a few weeks of full sky coverage, measuring $\lesssim 200$ GRBs.

3.3. Number – Peak flux distribution: with evolution

3.3.1. Frequency Density Evolution

In Figs. 5 - 7, we show the results of comparison of the observed BATSE number – peak flux distribution with distributions calculated including evolution in the frequency density of GRBs for $-4 \leq p \leq 4$. As in Fig. 3, the solid lines are constant KS probability (1%, 10% and 33%) that the assumed model could produce a distribution as or more disparate than that observed. The broken lines are constant n_* (see Sec. 3.1.). In panels a, b, and h of each figure, the constant n_* lines are marked in units of GRBs per L^* galaxy per 10^6 yr and are identical to the lines in the other panels.

In the case of “negative” density evolution ($p < 0$; panels b, d, f, and h), stronger evolution has the effect of decreasing the implied z_{\max} values. If the number density decreases with increasing $(1+z)$, lower values of z_{\max} are required; as a result, higher values of n_* are required, increasing by a factor of ~ 10 as

p decreases from -1 to -4 . This behavior is qualitatively the same independent of the assumed spectrum (of those considered here). The observed number – peak flux distribution is consistent with strong negative density evolution up to $p=-4$, but requires a lower z_{\max} and a higher n_* (by $\times 10$) than a zero-evolution scenario.

In the case of “positive” density evolution ($p > 0$; panels a, c, e, and g), stronger evolution has the effect of increasing the implied z_{\max} values. For $p > 2$, the models could not always be reasonably fit to the observed distribution for values of $z_{\max} < 1000$, and exhibit dependency on the assumed spectrum. The z_{\max} values are strongly dependent on the assumed source spectrum, with the range of acceptable z_{\max} values increasing greatly for lower values of Ω_0 (< 1).

3.3.2. Peak Luminosity Evolution

Figs. 8 - 10 show the results of comparison of the observed BATSE number – peak flux distribution with distributions calculated including evolution in the peak luminosity of GRBs with $(1+z)$ according to Eq. 2. As in Fig. 3, the broken lines are constant KS probability (1%, 10% and 33%) that the observed number – peak flux distribution could have been drawn from the model distribution. The solid lines are constant n_* . In panel a of each figure, the constant n_* lines are marked in units of GRBs per L^* galaxy per 10^6 yr at the present epoch, and are identical to the lines in the other panels.

In the case of “negative” luminosity evolution ($l < 0$; panels b, d, f, and h), stronger evolution has the effect of decreasing the implied z_{\max} values, while increasing the n_* values. The most probable values of z_{\max} decrease from the range of ~ 1.0 - 2.2 (no evolution) to ~ 0.3 - 0.7 ($p=4$), and the range of implied n_* increases by a factor of 10 (~ 0.3 - 5 to 3 - 50 GRBs per L^* galaxy per 10^6 yr at the present epoch).

In the case of “positive” luminosity evolution ($l > 0$; panels a, c, e, and g), stronger evolution increases the value of z_{\max} , and also constrains the value of Ω_0 from above. Thus, for models with $l > 0$, the observed number - peak flux distribution distorts the acceptable parameter space considerably from that of zero evolution models; the acceptable parameter space becomes a strong function of both z_{\max} and Ω_0 (see Figs 8g, 9c, and 10c). Acceptable parameter space is found for l as high as 3, (complex spectrum or $\alpha=2.5$) (Figs. 9c and 10c).

3.4. Relative Time Dilation

For values of z_{\max} and Ω_0 which produced models with KS probabilities $> 1\%$, we also found the ratio R_{rel} :

$$R_{\text{rel}} = (1 + z_{\max}) / (1 + z_{\min}) \quad (20)$$

the ratio of the maximum redshift of GRBs in the sample to the minimum redshift of GRBs in the modeled 394 GRB sample. This ratio is the maximum amount of relative time dilation (or relative energy shift) between GRBs within the observed sample. In models with no source evolution, this ratio is very close to $(1+z_{\max})$. However, when strong positive frequency density evolution is present ($p > 1$), this ratio was usually much smaller than $(1+z_{\max})$. This is not unexpected, as frequency density must change sharply in

$(1+z)$, requiring that the bursts be localized within a small range of redshifts. For all spectra considered, the ratio R_{rel} is always $\lesssim 9$ (for all models with $\Omega_0 < 1$ and KS probability $> 10\%$) even for z_{max} values in the range of 100-1000.

In Fig. 11, panels a, b, and c, we show the 1% KS probability contours for the models for each assumed spectrum with positive frequency density evolution which still had acceptable parameter space at $z_{\text{max}} < 1000$ (taken from Figs. 5e, 6c, and 7c). In all cases, the values of R_{rel} are smaller than $(1+z_{\text{max}})$, indicating that all 394 observed GRBs (in these models) come from $z_{\text{min}} > 1$. The very shallow spectrum $\alpha = 1.0$ requires a range of R_{rel} which is relatively high (30-60) while the more realistic complex spectrum, as well as the $\alpha = 2.5$ spectrum, has relatively low values of R_{rel} (5-8) for models which place the faintest observed BATSE bursts at $z_{\text{max}} = 100-1000$.

We show similar results in Fig. 11d-f for the models with positive peak-luminosity evolution (taken from Figs. 8g, 9c, and 10c). In all cases, the value of R_{rel} is, at lowest, only a factor of a few lower than z_{max} , requiring large amounts of relative time-dilation for bursts at high values of z_{max} (factor of 20-200 for $z_{\text{max}} \sim 20-800$). Models with relatively small amounts of peak luminosity evolution ($l=1-2$), place the faintest GRBs at $z_{\text{max}} \sim 10$ with $R_{\text{rel}} \sim 10$.

4. Discussion and Conclusions

We have examined the observed $\log N - \log F_{\text{peak}}$ distribution of 394 GRBs observed by BATSE, and, assuming a cosmological model with a constant number of bursts per comoving volume per comoving time, the distribution is consistent with the “standard candle” GRBs with peak fluxes > 0.3 phot $\text{cm}^{-2} \text{sec}^{-1}$ originating at $z_{\text{max}} = 0.8-3.0$ (90% confidence), with the most likely values in the range of 1.0-2.2, largely insensitive to the assumed Ω_0 .

In performing this analysis we assumed that consistent model parameter space of the observed GRB sample could be bracketed by the single power-law spectral models ($\alpha=1.0$ and 2.5) which, themselves, “bracket” the GRB broken power-law spectra fit by Band et al. (1993) (see Fig. 2). We find this assumption to be justified, as the dominant portion of the Band et al. (1993) spectra fall between these two single power-law spectral models, and the behavior of the complex spectrum also fell between that of the two single power-law spectral models. We presumed that the broken power-law parameter E_b is constant (that is, is measured to be $E_b/(1+z)$ from a burst at the redshift $1+z$) and that a dominant number of bursts have an $E_b \sim 300$ keV. In the worst possible case of inconsistency with this assumption – that E_b varies randomly – the behavior of the models will be bracketed by behavior of the broken power-law spectrum at its extremes, which behaves like a single power-law spectrum of $\alpha \gtrsim 1.0$ or $\lesssim 2.5$.

Our choice for the the value of E_b in our simulations was made to maximize the effect of the difference between the “complex” spectrum and the power-law spectrum. This was done to see investigate if the power-law approximation models produce grossly different results from the more observationally correct “complex” spectrum. We find that the behavior of the “complex” spectrum model is within the limits of the behavior set by the two single-power-law spectrum models. This is not unexpected; if the redshifts over which the bursts span are modest in extent, then the broken power-law model behavior is dominated by the low-end of the spectrum (*i.e.* $\alpha_b = 1.0$; Eqn. 3), while if the bursts span over a large range of redshifts, the broken power-law model behavior is dominated by the the high-end of the spectrum (*i.e.* $\beta = 2.5$; Eqn. 4).

We find that, for the zero-evolution models, strong limits may be placed on the GRB rate at 0.3-5.0 GRBs per L^* galaxy per 10^6 yr, for $\Omega_0 \lesssim 1$. This is consistent with the values found by MP for an $\Omega_0=1$ universe (although MP assumed flatter energy spectra than has been observed by BATSE ; $\alpha=0.5-1.5$ vs 1.5-2.5) and with those found by Piran (1992).

When this work was largely complete, similar analyses by Cohen & Piran (1995) considering non-evolving cosmological models using the 2B BATSE data base came to our attention. Our conclusions are consistent with their analyses.

We find that, for BATSE, $\gtrsim 4000$ (most likely $\sim 9,000$) GRBs are required to use $\log N - \log F_{\text{peak}}$ to constrain a zero-evolution, standard-candle peak luminosity cosmological model parameter Ω_0 to between 1.0 and 0.1, and $\gtrsim 9000$ to constrain Ω_0 to between 0.5 and 0.1. This requires a minimum of 6 yrs of BATSE integration; a duty cycle of 50% places this above the projected mission lifetime of BATSE.

The observed number – peak flux distribution can accommodate both peak luminosity evolution of GRBs and frequency density evolution.

The frequency density evolution, if any, can be accommodated by a power law in $(1+z)^p$ for the ranges of z_{max} and Ω_0 considered here. We find no acceptable models for $p \gtrsim 3$.

If we assume the difference in timescales (by $\times 2$) between bright and faint bursts found by Norris et al. (1994a) and Wijers & Paczyński (1994) is due to the relative time dilation of bursts within the sample, then we find acceptable model-parameter space which would place the faintest GRBs observed by BATSE at z as high as 1000 with maximum relative time dilation of $R_{\text{rel}} \sim 8$. As the aforementioned studies necessarily average over a substantial fraction of the brightest and faintest bursts, the relative time dilation they measure should be somewhat smaller than our parameter R_{rel} (which compares only the brightest and faintest single bursts). We find that values of R_{rel} as high as ~ 10 are consistent with the measurements of the factor of two difference in timescale between bright and faint bursts.

The $p=1$ (frequency density evolution) model is identical to the assumed GRB model used by Wickramasinghe (1993) (which they proposed as a non-evolving model). For $\Omega_0=1$ in this model, we find higher values of the most probable z_{max} (1.7-3.0 vs. 0.5-1.7; comparing with their Fig. 2, at the completion limit). This is probably due to the fact that we use a lower flux limit (and thus, “see” to greater redshifts); when we perform the same analysis to the 99% flux limit of BATSE (the flux limit used by Wickramasinghe et al.), we find z_{max} in the range 0.9-1.7, similar to that found by Wickramasinghe (1993).

We have found that the strongest positive frequency density evolution scenario ($p=2$) with acceptable model parameters for all spectra considered permit z_{max} to be as great as ~ 200 , with values of R_{rel} as high as 9 for hard bursts, and in the range of 2-5 for softer bursts. When we consider that many GRBs of different spectra were averaged over by Norris et al. (1994a) and Wijers & Paczyński (1994) to obtain the timescale difference of a factor of 2 between bright and faint bursts, we estimate that the $p=2$ scenario cannot be excluded on this basis; averaging together harder GRBs with softer GRBs diminishes the higher values of R_{rel} , and the averaging together of many bursts of different peak fluxes (and thus, at different redshifts) diminishes the amount of relative timescale difference within the burst sample to an level consistent with these observations.

Application of Eq. 9 to the $p=2$ scenario shows that the implied photon luminosities in the redshifted passband at the epoch of the source are a function of α , z_{max} , and Ω_0 , and can be different for different assumed spectra by several orders of magnitude or identical to within a factor of order unity, dependent on the assumed Ω_0 . For example, for an assumed spectrum of $\alpha=1.5$, the $p=2$ scenario places

statistically acceptable (KS prob $> 1\%$) populations at (z_{\max}, Ω_0) of about (10, 1.0), (60, 0.2), and (200, 0.06). For these values of (z_{\max}, Ω_0) the ratio of photon luminosity to flux is 4×10^{59} , 2×10^{62} , and 7×10^{64} (all in units of $\text{cm}^{-2} h_{80}^{-2}$). The implied source luminosity to flux ratio of non-evolving population, with a source at z_{\max} of 1.5 for $\Omega_0=1$ is $6 \times 10^{57} \text{ cm}^{-2} h_{80}^{-2}$. Thus, the evolutionary scenarios require sources with much greater luminosities, by up to 7 orders of magnitude) than non-evolving scenarios.

Photons with energies as high as 10 GeV have been observed from some GRBs (Schneid et al. 1992; Kwok et al. 1993; Dingus et al. 1994; Sommer et al. 1994). The Cosmic Microwave Background Radiation is capable of scattering photons at such energies when they are emitted from very high redshifts (Babul, Paczyński & Spergel 1987). There is essentially a “wall” in redshift-space to photons of a given energy due to scattering by the CMBR, such that photons of energy E MeV will not be observed if they are emitted beyond a redshift $z \sim 7400(E/1\text{MeV})^{-0.484}$ (Babul, Paczyński & Spergel 1987). Thus, photons of energy 1 GeV would not be observed from a GRB if the GRB occurred at a redshift greater than ~ 260 . Photons of such energies have been detected from only a very small fraction of the observed GRBs, and these GRBs are typically among the brightest observed, which may come from redshifts a factor of ~ 5 -8 lower than the maximum redshift; for example, if $z_{\max}=1000$, the brightest GRBs would come from redshifts 125-200, and would thus these high energy photons would not be scattered by the CMBR. Thus, the observation of some such photons is not inconsistent with BATSE seeing GRBs from redshifts as great as 1000.

The implied comoving burst rate in the strong ($p=2$) frequency density evolution scenario is $\times 100$ higher for hard bursts ($\alpha=1.5$) than for soft bursts ($\alpha=2.5$). For weaker ($p=1$) frequency density evolution, the comoving burst rate of harder GRBs is only $\sim \times 10$ greater than that of the softer GRBs. These high spectrally-dependent differences in the relative comoving burst rates are a constraint on models which contain strong GRB frequency density evolution. Further, ‘standard-candle’ models which contain strong frequency density evolution require that the GRBs of different spectra either have different number–peak flux relations, or that the relative burst rates conspire with the evolution rate and space-time geometry in the epoch where the majority of GRBs of a particular spectrum occur to produce identical number–peak flux distributions, independent of the intrinsic source spectrum. In Fig. 12, we show the cumulative distributions of the 100 spectrally hardest and 100 spectrally softest GRBs from the present sample; a 2-distribution KS test shows them to be statistically identical (KS probability 59%).

The peak luminosity evolution, if any, can be accommodated by a power law in $(1+z)^l$. For the peak luminosity evolution models, the relative time dilation between the faintest and brightest bursts (R_{rel}) can be as high as 200 for the models considered here, and is very roughly between $(1+z_{\max})/4$ and $(1+z_{\max})$, favoring the high value for low z_{\max} , but decreasing to the low value for high z_{\max} . With the estimate that R_{rel} must be $\lesssim 10$ to be consistent with the observed amount of relative time dilation within the GRB sample, then the present analyses excludes luminosity evolution models which place the faintest observed GRB in the present sample at $z_{\max} \gtrsim 10$. If we also require, as we have in the frequency density evolution, that the luminosity evolution scenarios permit relative time dilation of at least a factor of 2, then we find that that negative luminosity evolution cannot be stronger than $l=2$.

We conclude that the Number-peak flux distribution of GRBs observed by BATSE is consistent with a homogeneous, zero evolution source population, to a z_{\max} most probably in the range 1.0-2.0 for GRBs with peak luminosities of $\sim 0.3 \text{ phot cm}^{-2} \text{ sec}^{-1}$, and that the flattening observed in this distribution is consistent with being due only to cosmological effects. The amount of frequency density evolution of the form given by Eqn. 12 is constrained, with the exponent $p < 2$. The amount of peak luminosity evolution of the form given by Eqn. 7 is constrained, with the exponent $l < 2$.

As the constraints on z_{\max} , p , and l , improve with greater numbers of GRBs, we expect similar future work with larger samples to be useful in the parameterization of the GRB source population. However, to put practical limits on GRB cosmological models, much deeper observations are required.

We wish to draw the reader’s attention to the fact that in considering one evolution model (frequency density evolution or luminosity evolution), that the other evolutionary model was held to be zero, which may not be the actual observational case. Given the wide range of acceptable parameters for even this case, it is not at all clear that it is, even in principle, possible to jointly constrain all these different functions and values (frequency density evolution, luminosity density evolution, z_{\max} , Ω_0 , n_*) with the single observational $\log N - \log F_{\text{peak}}$ curve. We also wish to point out that, as we have assumed power-law forms of the evolution, the validity of these conclusions is limited to actual evolution which follows power-law form.

If flattening of this distribution from the expected (from Euclidean geometry and spatial homogeneity) $-3/2$ power law are due only to cosmological effects, then BATSE will not integrate sufficient numbers of GRBs with peak flux $0.3 \text{ phot cm}^{-2}\text{sec}^{-1}$ to constrain Ω_0 to the range 0.1–1.0 during its projected lifetime, which would, with certainty, permit confrontation with cosmological models. However, Hakkila et al. (1994) estimates that BATSE can integrate enough GRBs in its lifetime to permit confrontation with a variety of galactic models. A mission with much fainter flux limits ($\times 70\text{--}400$), fortuitously pointed toward M31, could permit both after integrating for a period of months.

We find that the results of the present analysis is supporting, but not compelling, evidence of a cosmological origin for GRBs and much further work is required to investigate this hypothesis.

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Fig. 1.— Number vs. photon number power law slope α ; for 30 GRBs. Data was taken from Schaefer et al. (1994); bursts were selected for this sample which had peak photon fluxes greater than 4 photons $\text{cm}^{-2} \text{sec}^{-1}$ on the 64msec timescale – however, some “interesting” bursts were also included.

Fig. 2.— (a) The spectral models fit by Band et al. (1993) to 55 GRBs, normalized at 50 keV. We show these models for the energy range 50-5000 keV, although the BATSE GRB spectra largely span 20-2000 MeV, to show the extrapolated behavior of the spectra at higher energies. (b) Single power-law spectra with $\alpha=1.0$, and 2.5, and the complex spectrum (see text), normalized at 50 keV. These spectra define the limiting and average behavior of the observed spectra found by Band et al. (1993)

Fig. 3.— Results of comparisons of the BATSE observed number – peak flux curves with that expected from a constant number per comoving time per comoving volume source distribution, assuming different source photon spectra (complex, or single photon power law $\alpha=1.0$, 2.5; marked in each panel), as a function of Ω_0 , and the redshift of the faintest bursts observed (z_{max}). The solid lines are constant KS probability (1%, 10%, and 33%; note that lower KS probability lines enclose higher KS probability lines – the outer two lines are 1% and the inner two lines are 33%). The broken lines are the constant n_* , the number of bursts per L^* galaxy per 10^6 years at the current epoch, and are marked with their value.

Fig. 4.— Estimations of the required number of GRBs to differentiate (at the 99% confidence level) between two different values of Ω_0 , assuming standard candle GRBs and a constant number of GRBs per comoving time per comoving volume, as a function of the value z_{max} , the redshift at which the standard candle GRB would produce a flux equal to the completion limit, assuming no evolution in the GRB population. Solid line is $\alpha=1.0$, broken line is $\alpha=2.5$. a) Number of GRBs needed to differentiate between $\Omega_0=1.0$ and 0.1, as a function of the z_{max} . b) Number of GRBs needed to differentiate between $\Omega_0 = 0.5$ and 0.1 as a function of z_{max} c) Coverage (in years 4π ster) required to integrate the number of GRBs needed to differentiate between $\Omega_0=1.0$ and 0.1. d) Coverage (in years 4π ster) required to integrate the number of GRBs needed to differentiate between $\Omega_0=0.5$ and 0.1.

Fig. 5.— Results of comparisons of the BATSE observed number – peak flux curves with that expected from a parent population evolving in number per comoving volume according to Eq. 12, assuming a source power law photon index slope of 1.0, as a function of Ω_0 , and the redshift of the faintest bursts observed (z_{max}). The solid lines are constant KS probability (1%, 10%, and 33%; note that lower KS probability lines enclose higher KS probability lines – the outer two lines are 1% and the inner two lines are 33%). The “wiggly”ness of the lines is due to the finite resolution of the probability calculation, and does not reflect real structure in the probability contours. The broken lines are constant in n_* , the number of bursts per L^* galaxy per 10^6 years at the current epoch, and are marked with their value.

Fig. 6.— This is identical to Fig.5, except a “complex” photon spectrum is assumed (see text).

Fig. 7.— This is identical to Fig.5, except a photon power law spectral index of 2.5 is assumed.

Fig. 8.— Results of comparisons of the BATSE observed number – peak flux curves with that expected from a parent population evolving in peak luminosity according to Eq. 5, assuming a source power law photon index slope of 1.0, as a function of Ω_0 , and the redshift of the faintest bursts observed (z_{max}). The solid lines are constant KS probability (1%, 10%, and 33%) that the observed Number – peak flux distribution can be drawn from the simulated distribution. The broken lines are constant in n_* .

Fig. 9.— This is identical to Fig.8, except a “complex” photon spectrum is assumed (see text).

Fig. 10.— This is identical to Fig.8, except a photon power law spectral index of 2.5 is assumed.

Fig. 11.— This figure shows the range of acceptable values of R_{rel} for selected models with KS probabilities $>1\%$. The broken lines are constant KS probability enclosing models at $>1\%$. The two solid lines in each panel are marked constant R_{rel} . The value of R_{rel} changes monotonically between the two solid contours. Each panel indicates the assumed spectrum and the model parameter which is different from zero (p for frequency density evolution, l for peak luminosity evolution).

Fig. 12.— Comparison of the cumulative distribution for the 100 hardest (determined by the ratio of the 100-300 keV photon fluence to the 50-100 keV photon fluence) and 100 softest GRBs with 1024msec peak photon flux $> 0.3 \text{ phot cm}^{-2} \text{ sec}^{-1}$. They are statistically identical, with a KS probability of 0.59

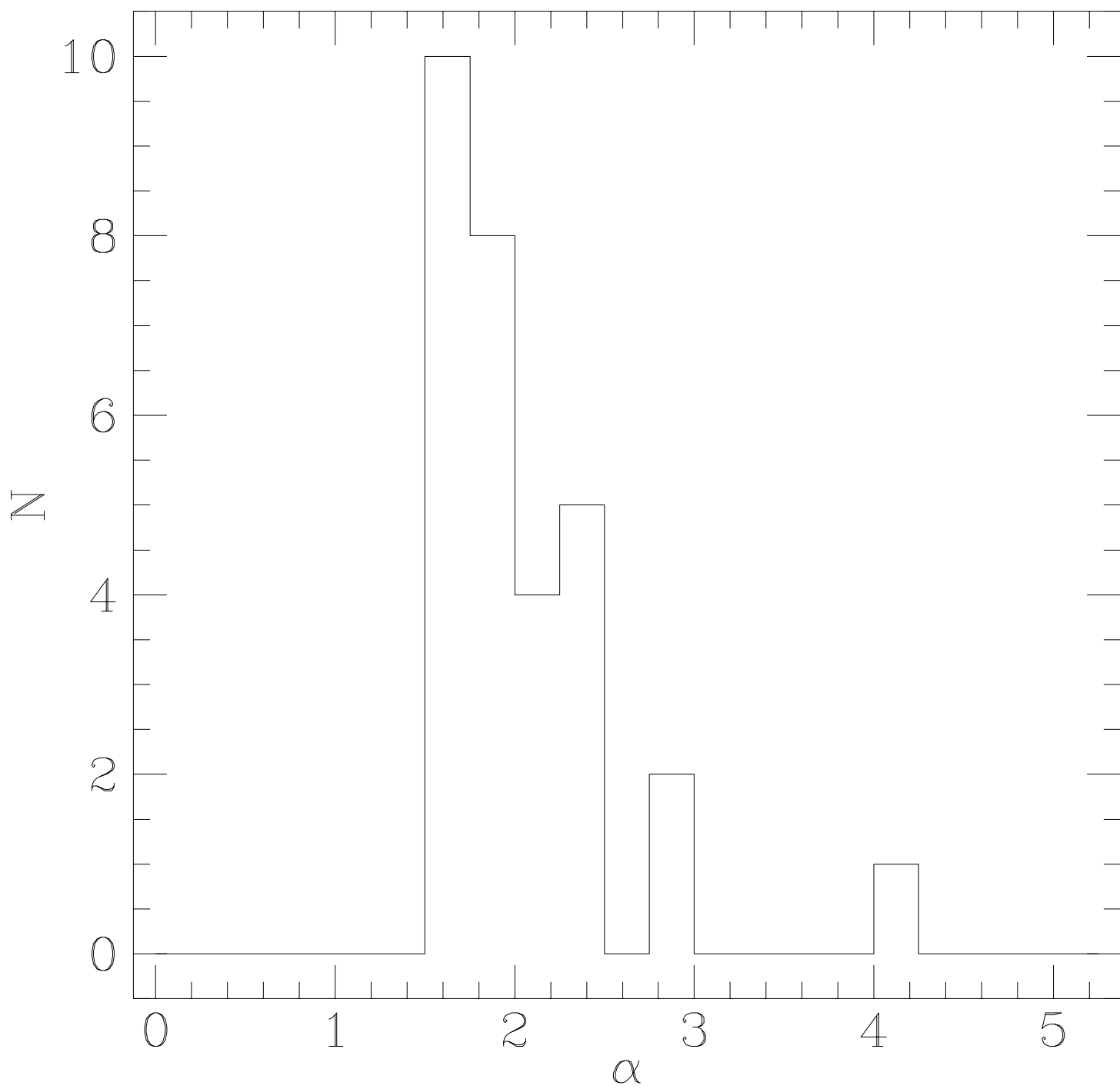


Figure 1

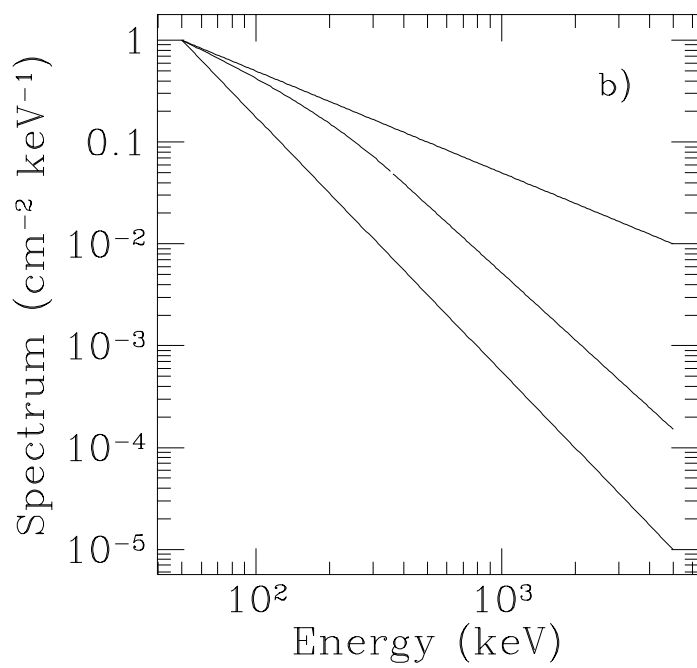
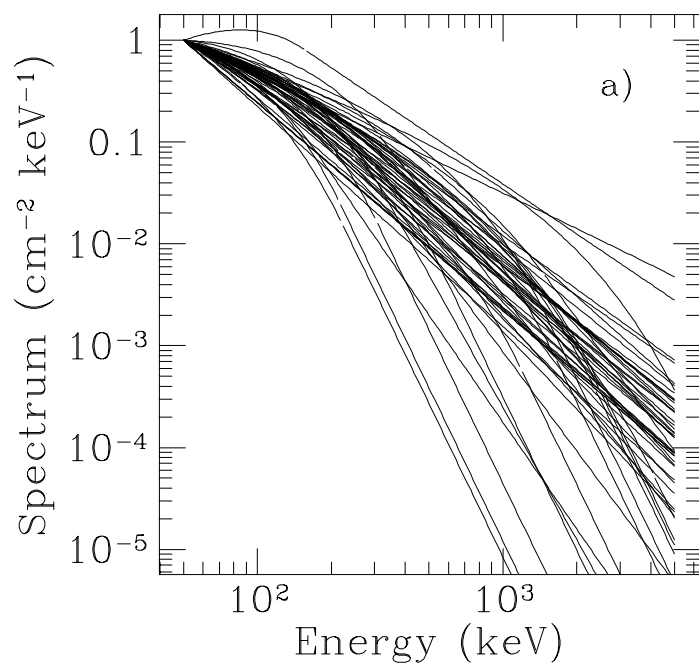


Figure 2

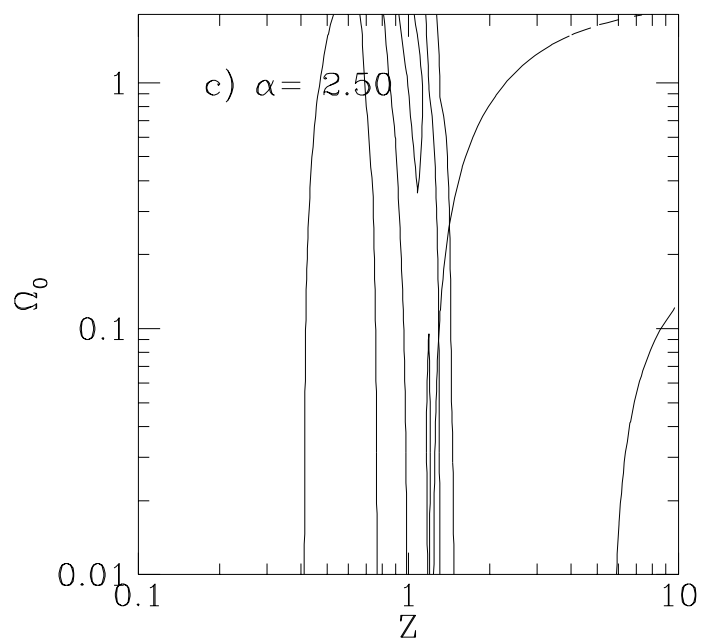
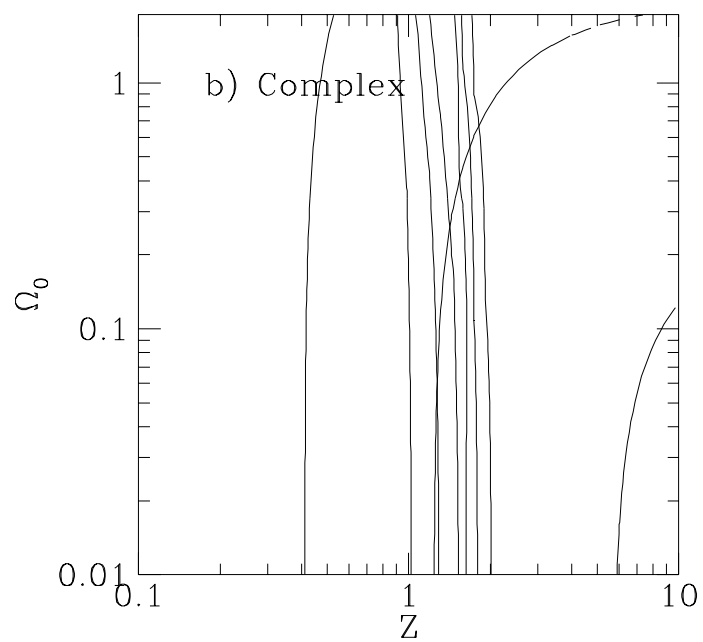
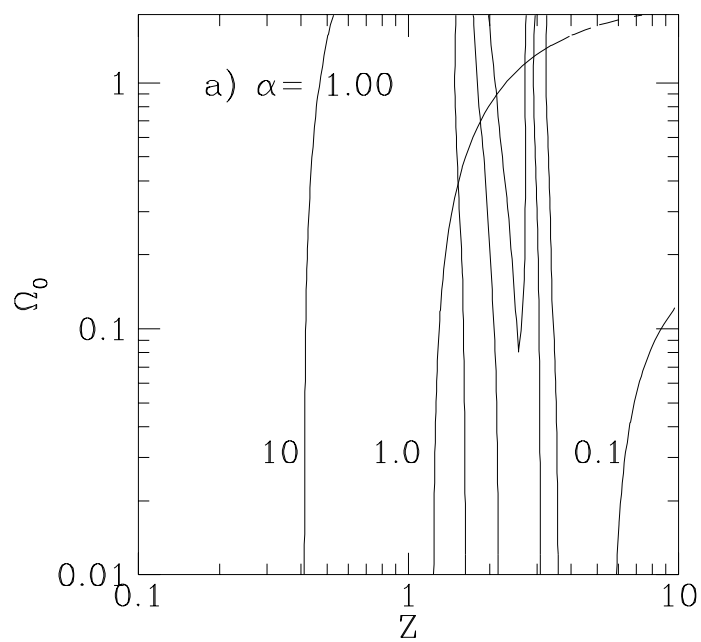


Figure 3

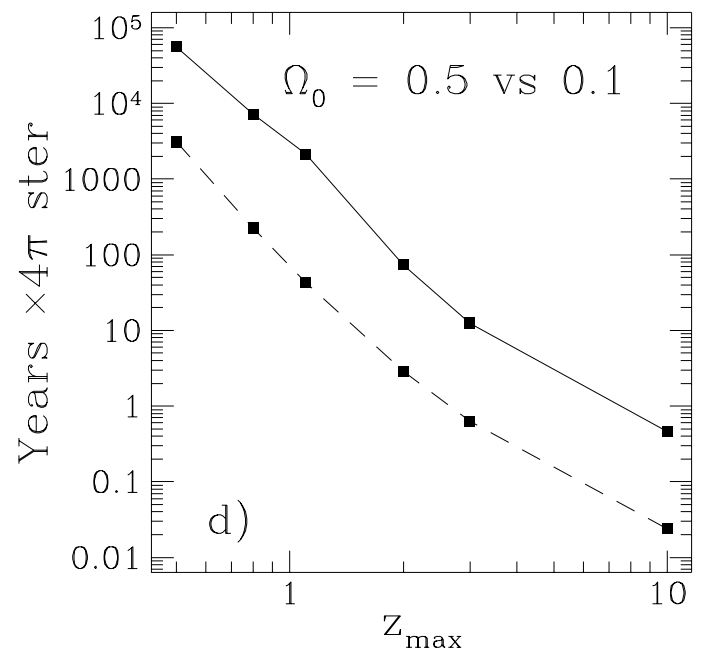
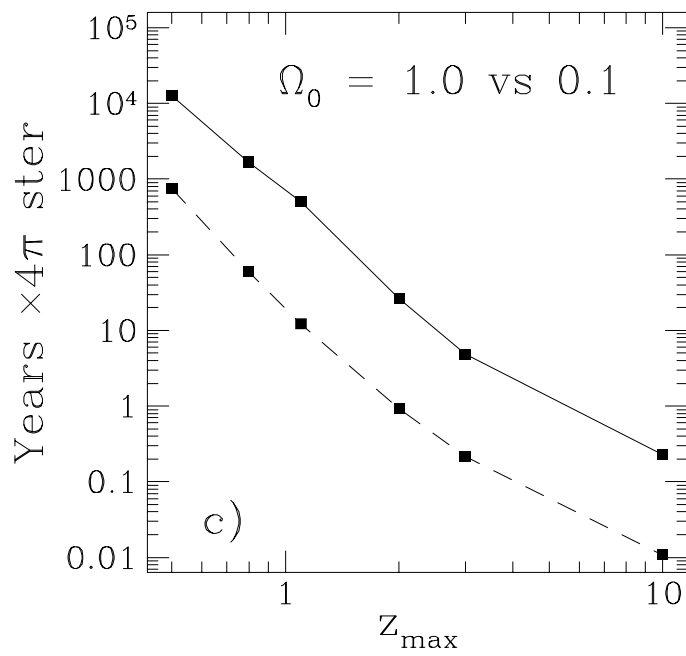
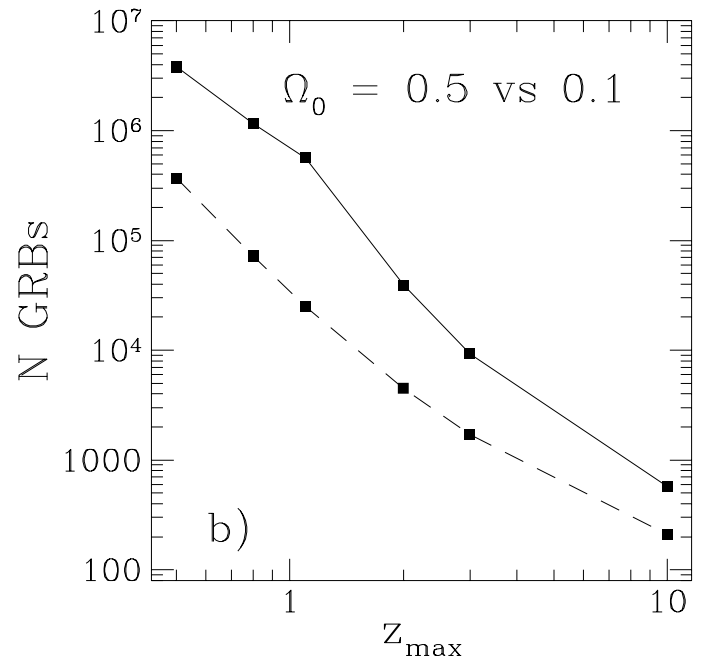
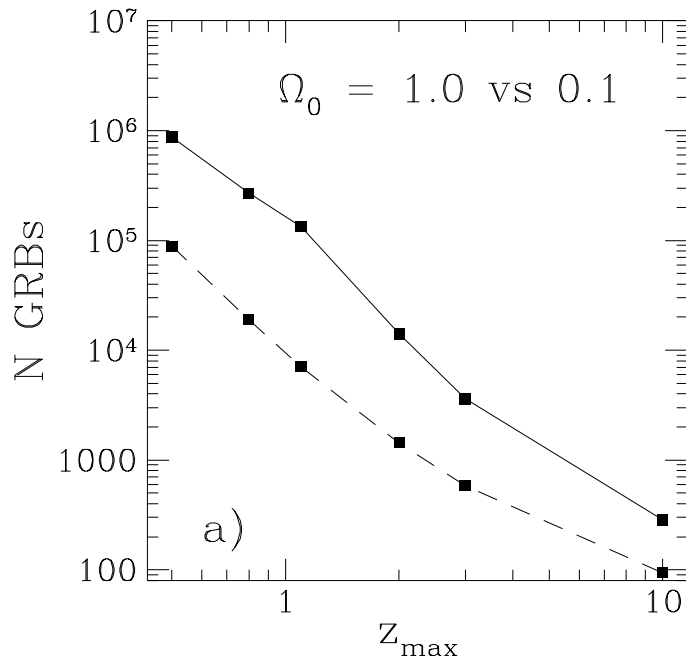


Figure 4

Density Evolution, $\alpha=1.00$

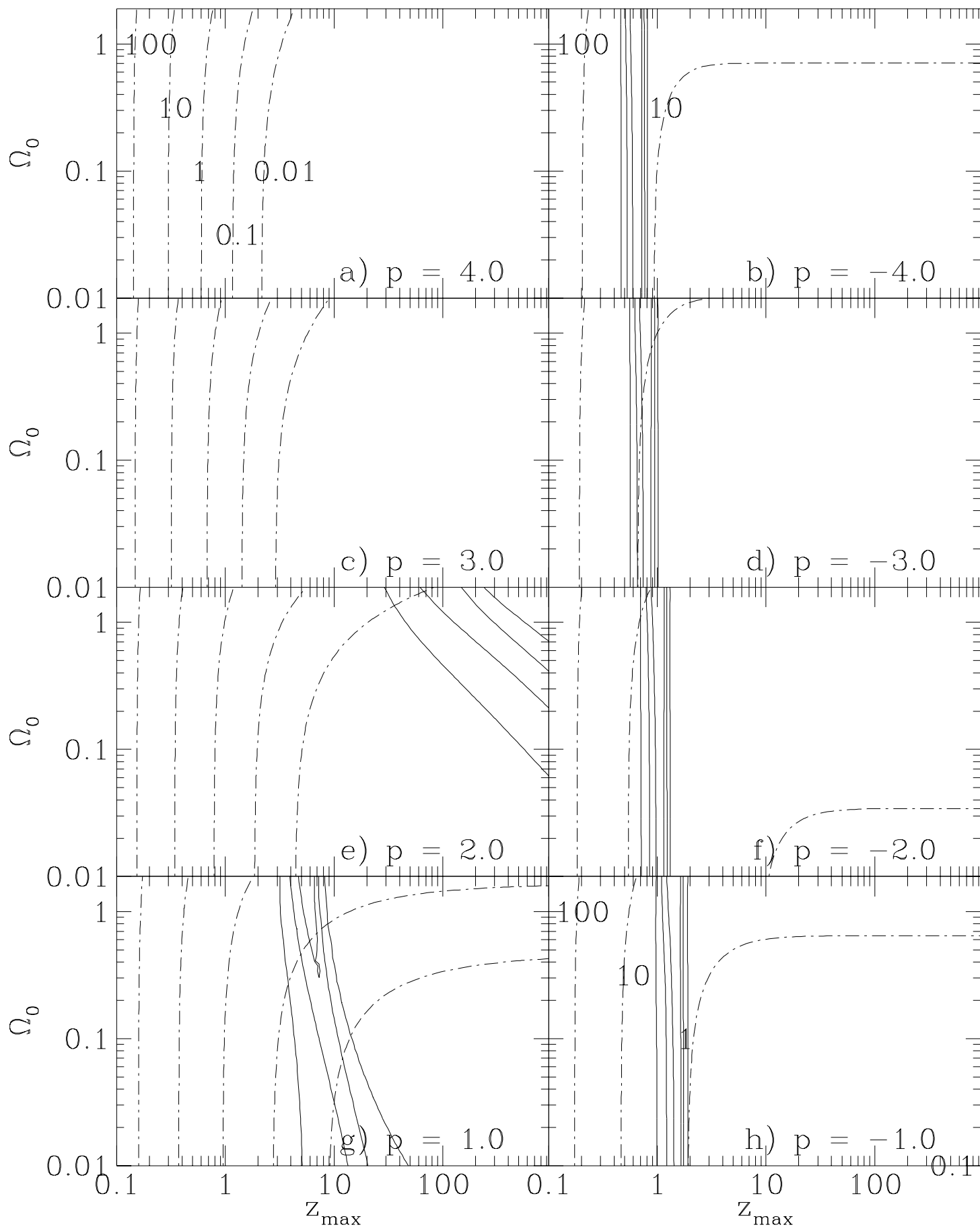


Figure 5

Density Evolution, Complex Spectrum

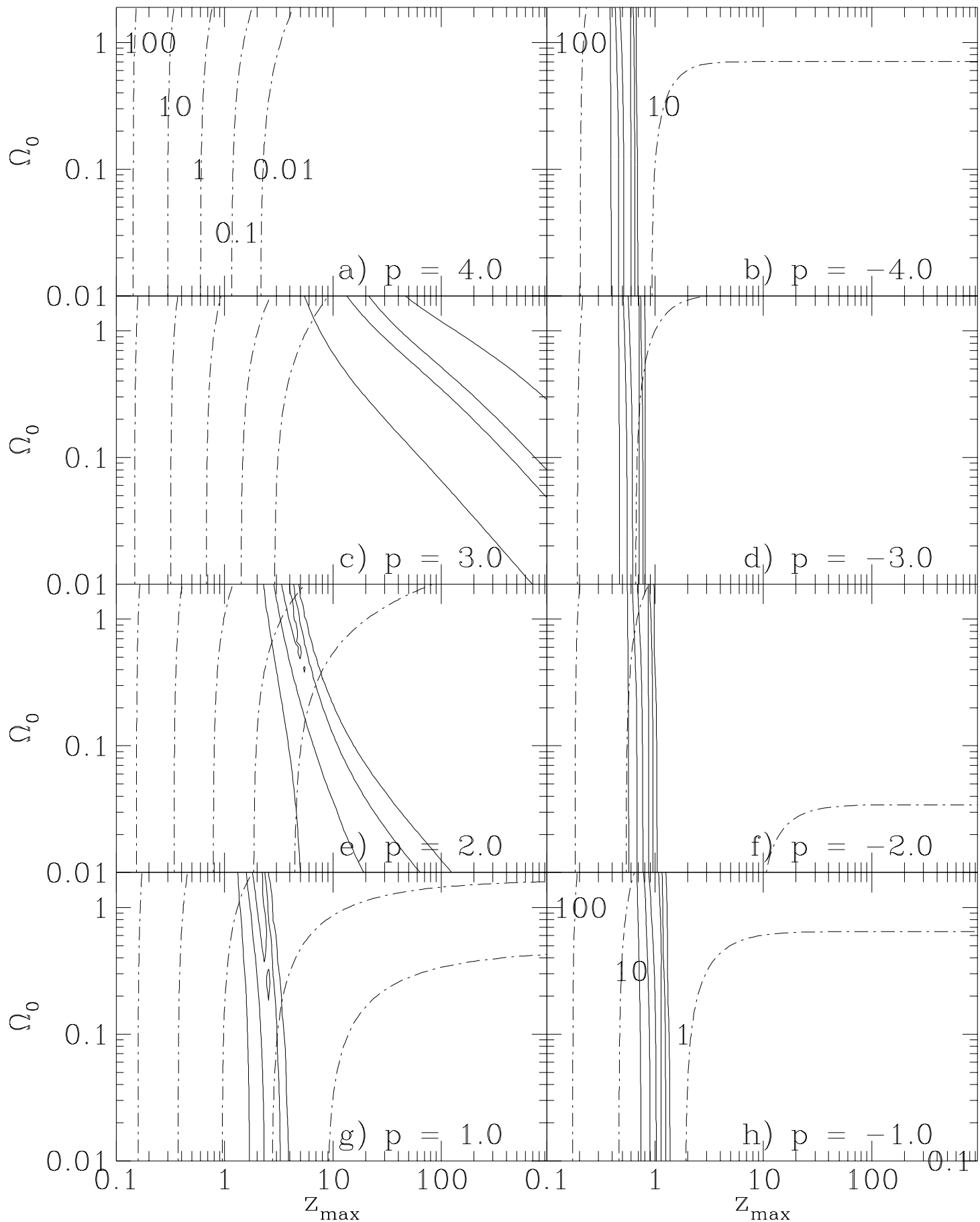


Figure 6

Density Evolution, $\alpha=2.50$

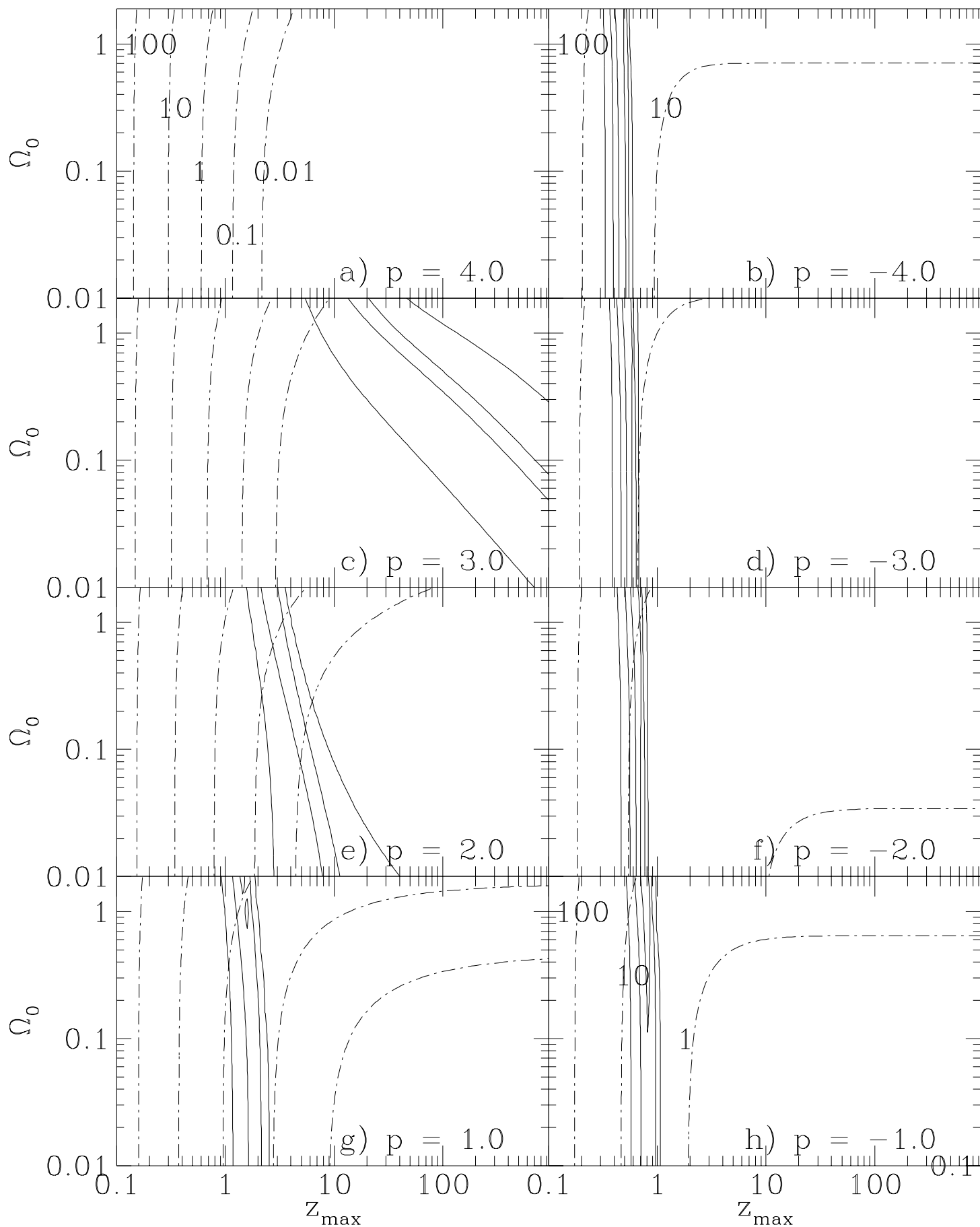


Figure 7

Luminosity Evolution, $\alpha=1.00$

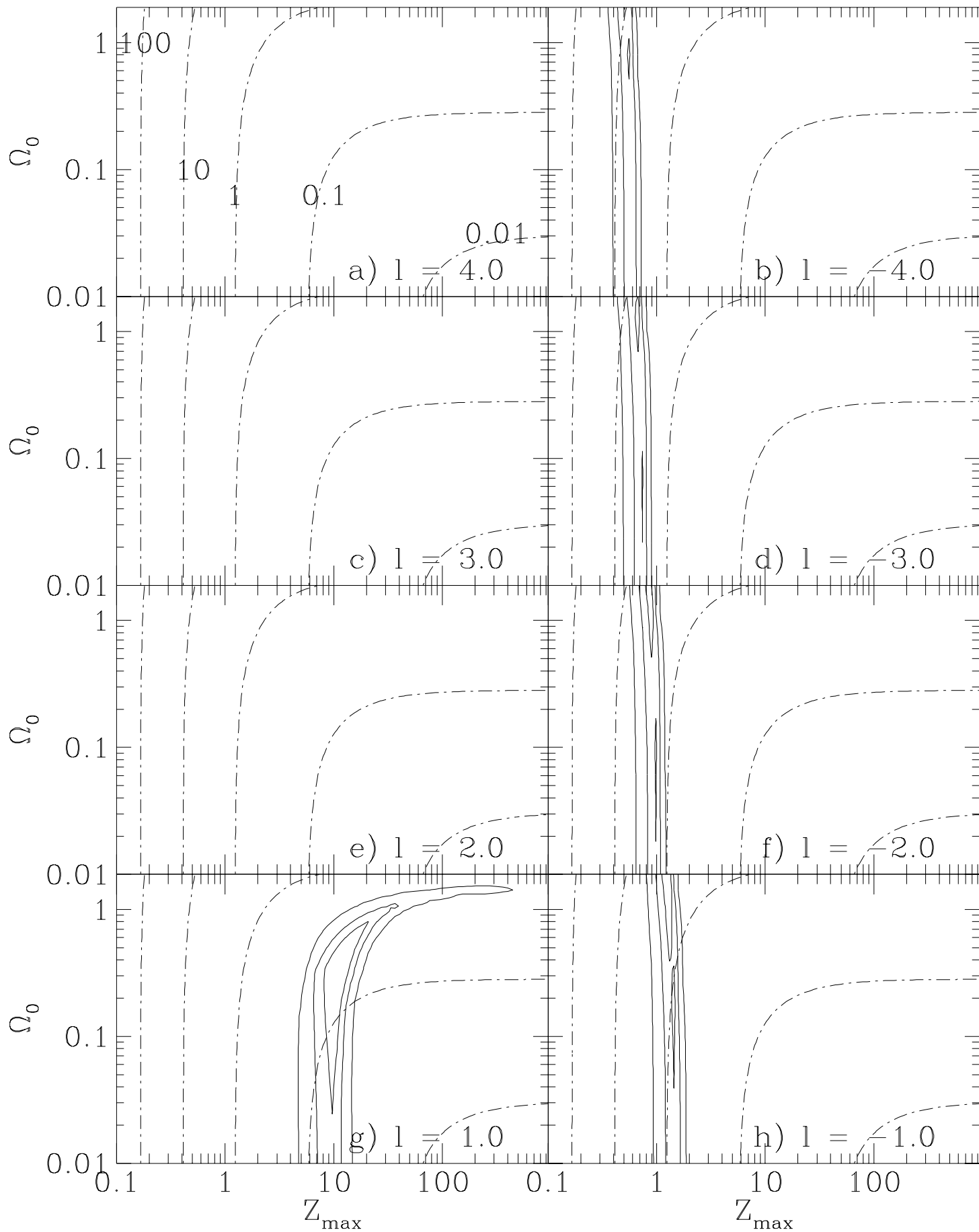


Figure 8

Luminosity Evolution, Complex Spectrum

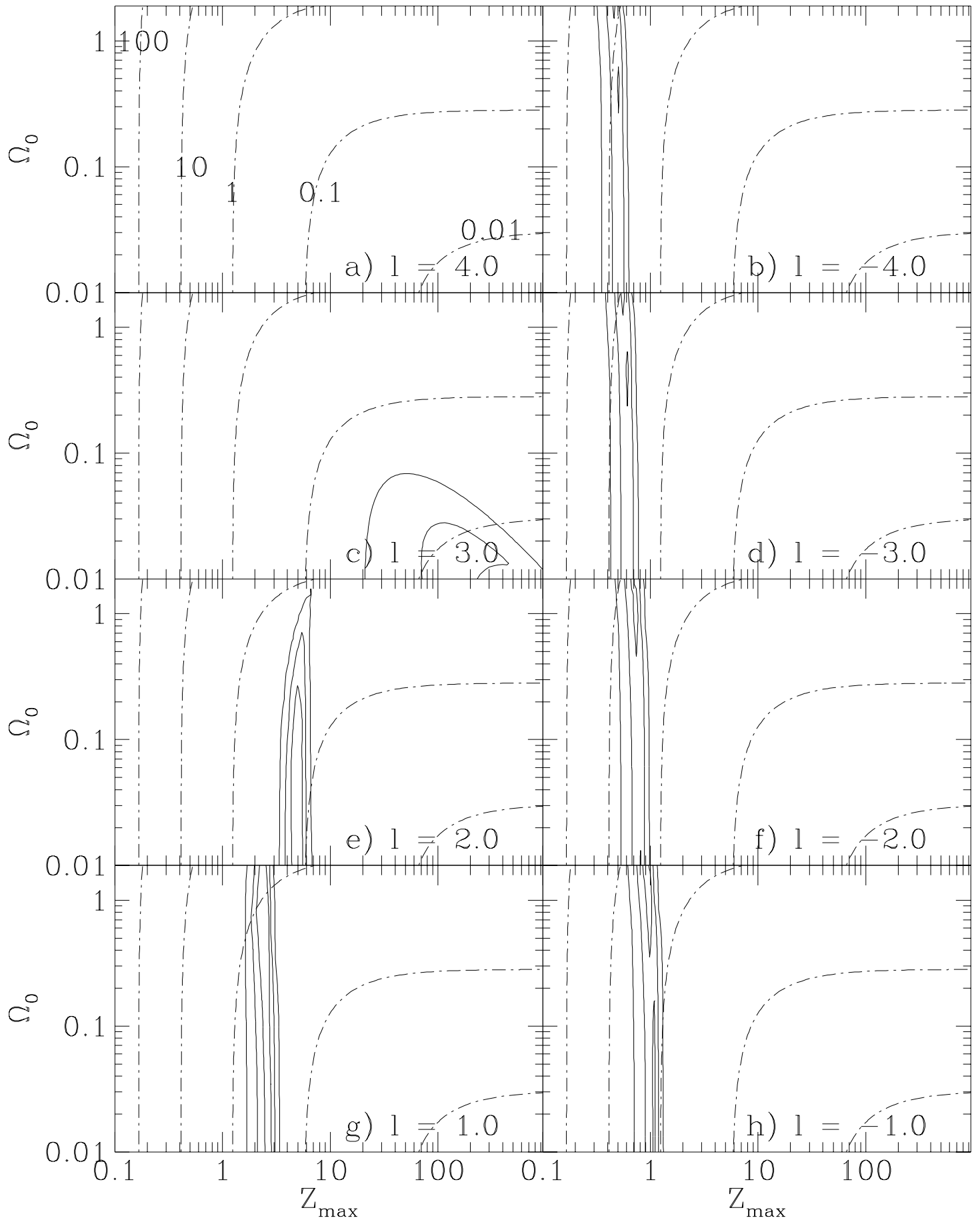


Figure 9

Luminosity Evolution, $\alpha=2.50$

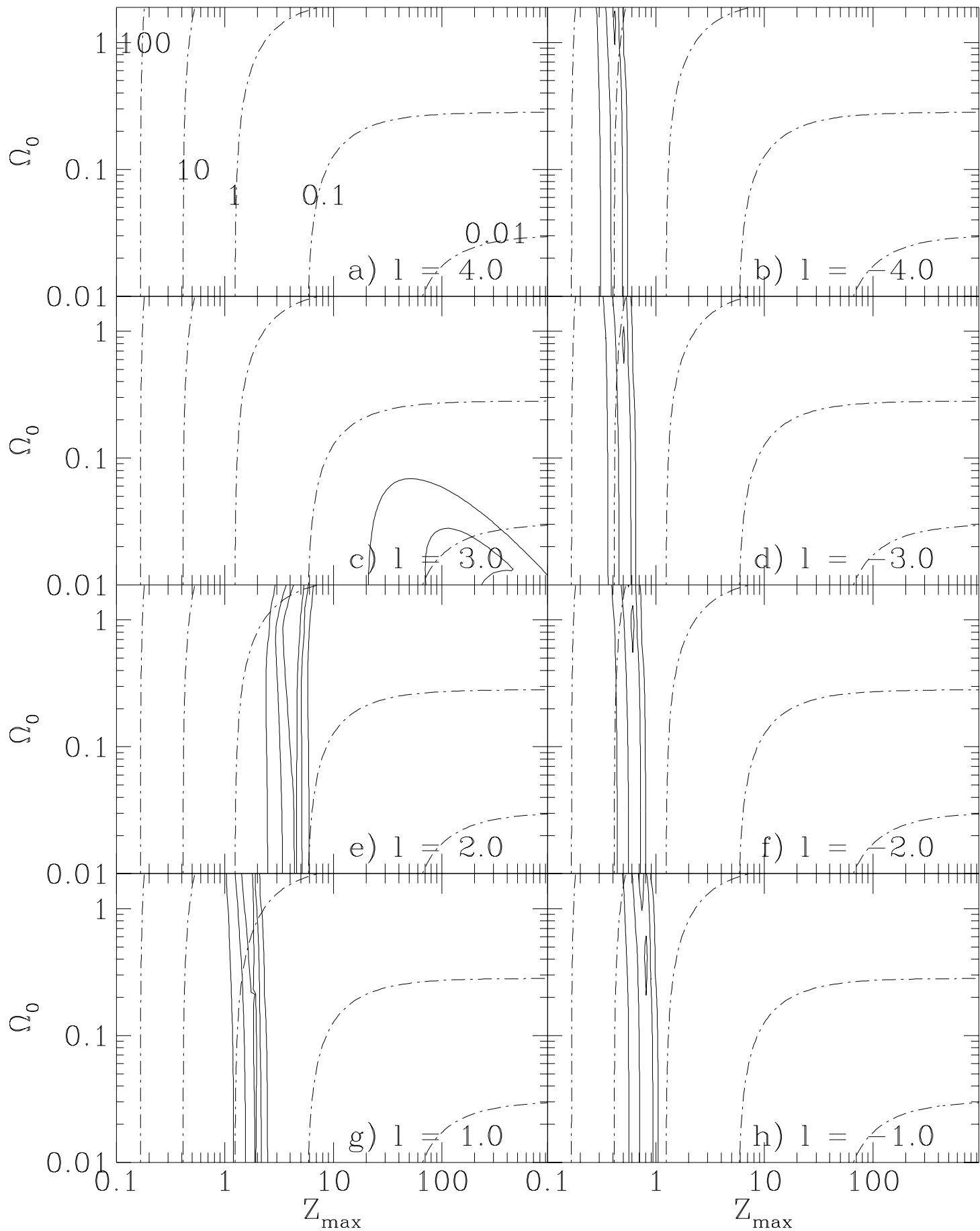


Figure 10

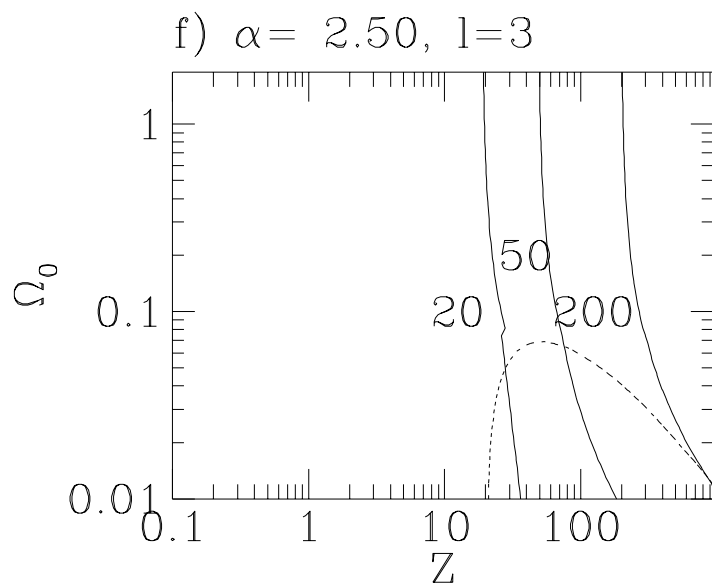
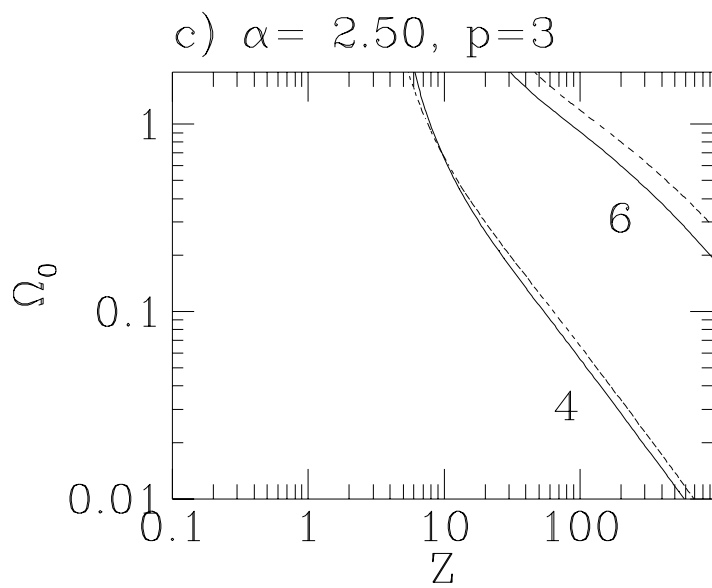
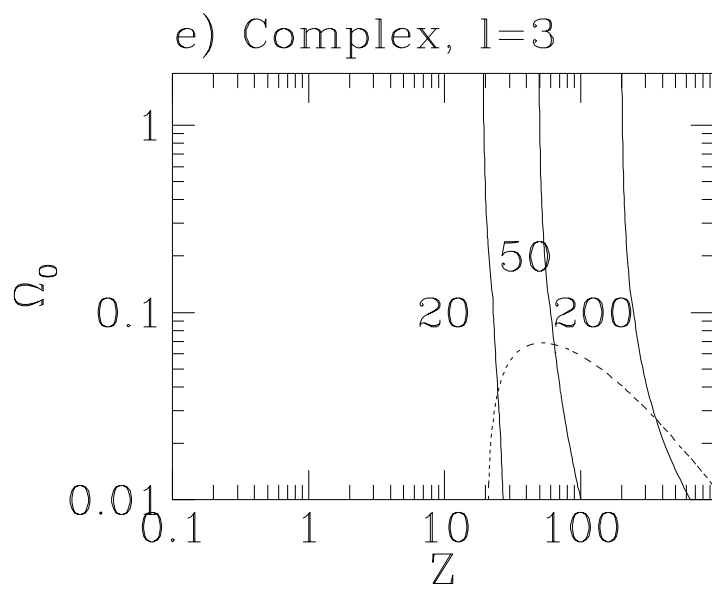
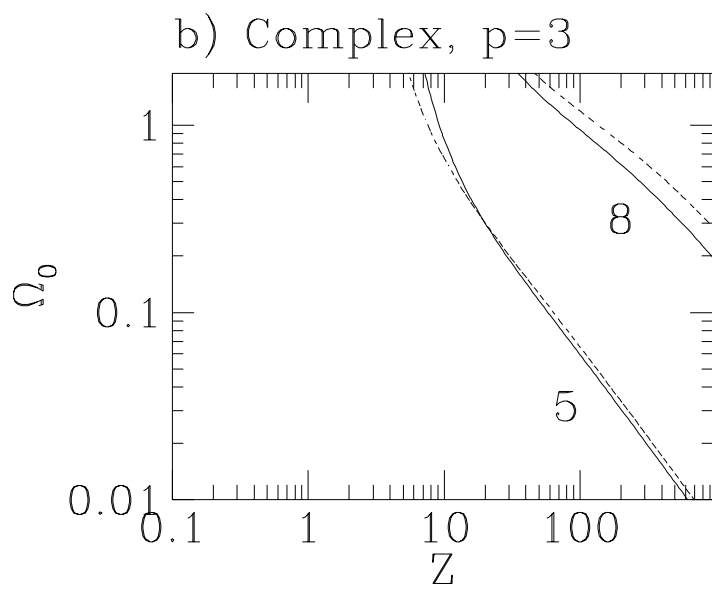
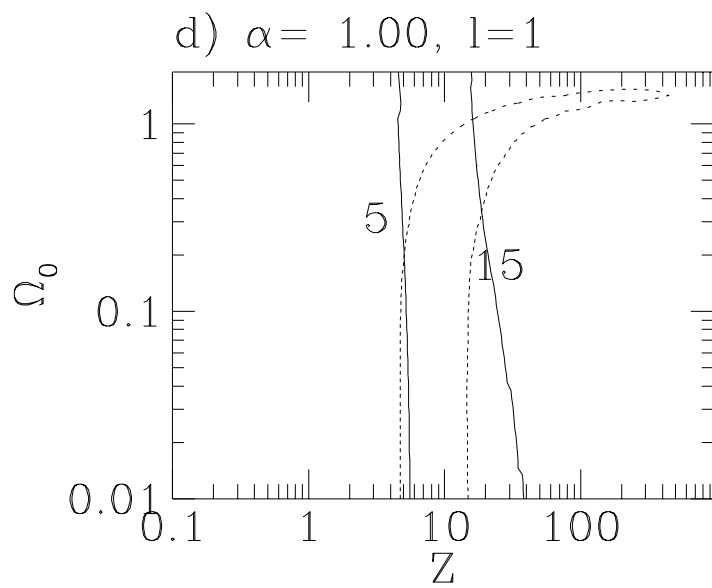
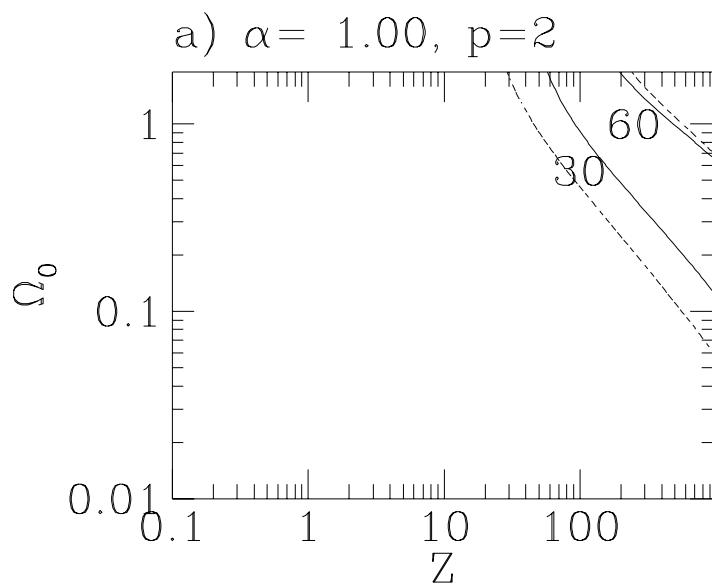


Figure 11

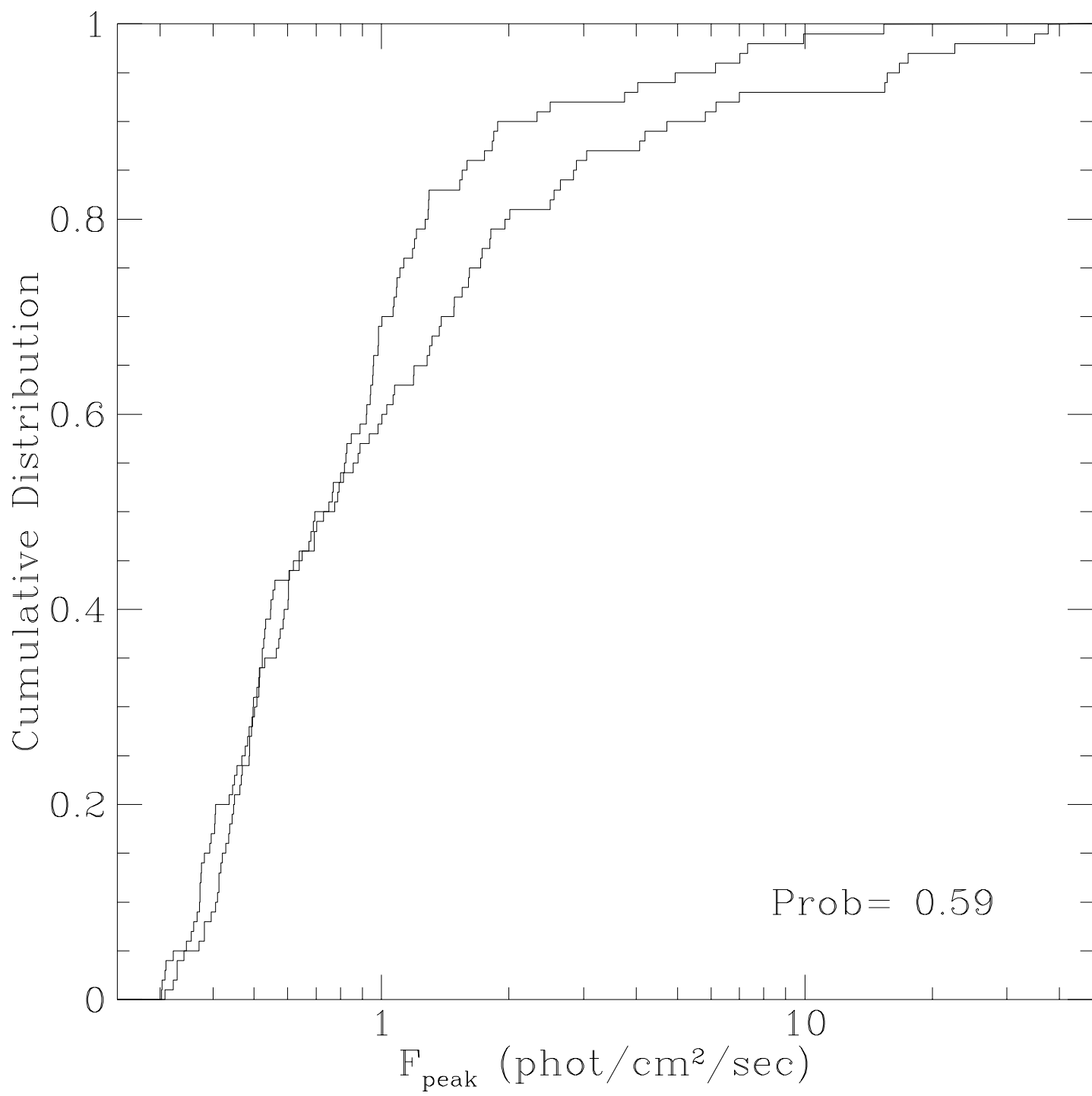


Figure 12